

Dynamic simulation of plate heat exchangers

CHITTUR CHANDRASEKHARAN LAKSHMANAN

BHP Melbourne Research Laboratories, 245-273 Wellington Road, Mulgrave 3170,
Victoria, Australia

and

OWEN EDWARD POTTER

Department of Chemical Engineering, Monash University, Clayton 3168, Victoria, Australia

(Received 22 December 1988 and in final form 13 July 1989)

Abstract—Previous models published in the literature are concerned with obtaining the steady state performance of plate heat exchangers using numerical solution of the differential equations describing the heat transfer process. The new numerical model, namely the 'cinematic' model developed by the authors has been used to simulate the dynamic behaviour of countercurrent systems such as fluidized beds (C. Lakshmanan and O. E. Potter, *Ind. Engng Chem. Res.* 26, 292-296 (1987)). This work reports the application of the 'cinematic' model to simulate the dynamic performance of plate heat exchangers. It is shown that the 'cinematic' model requires a very minimum amount of computations in accurately simulating the dynamics of plate heat exchangers. Also, it computes the dynamic and steady state profiles in one sweep, thus offering an easy and accurate approach to the design problems considered by others. It is also shown that representing a plate heat exchanger with an even number of channels by an equivalent true countercurrent system does not result in any significant errors as far as the steady state outlet temperatures are concerned.

INTRODUCTION

PLATE HEAT exchangers are commonly used in process industries for a variety of applications. A plate heat exchanger consists of a number of parallel flow channels formed by metal plates which are separated by gasket material around the perimeter of each plate. Nozzles for the flow of fluids extend through the frames to the plate packages. Heat is transferred through these plates from one fluid to the other (Fig. 1). A number of mathematical models of plate heat exchangers have appeared in the literature. These have been presented to solve both the design and performance problems. Watson *et al.* [1] and Jackson and Troupe [2] have used numerical integration techniques such as the classical Runge-Kutta methods. Buonopane *et al.* [3] have used the concept of dimensional analysis. The heat transfer process taking place in plate heat exchangers can be described by a system of differential equations. A thorough analysis of these differential equations is given by Wolf [4]. The analytical solution of this system of differential equations involves an expansion in terms of eigenvalues and eigenvectors characteristic of the system. The properties of the general mathematical model are published by Zaleski [5]. He has discussed the application of the methods to multiple channel exchangers without channel interconnections and to two fluid plate heat exchangers in which each of the fluids flows in every second channel. Marano and Jechura [6] have commented that Wolf [4] has not applied the technique to a plate heat exchanger and Zaleski [5] has only

considered relatively simple problems. Marano and Jechura [6] have presented a method suitable for digital computers to simulate the performance of plate heat exchangers. Their method does not require initial guesses for any of the outlet temperatures from the exchanger. The system of ordinary differential equations is expressed in matrix form, and their solution is expanded in terms of the eigenvalues and eigenvectors of the matrix.

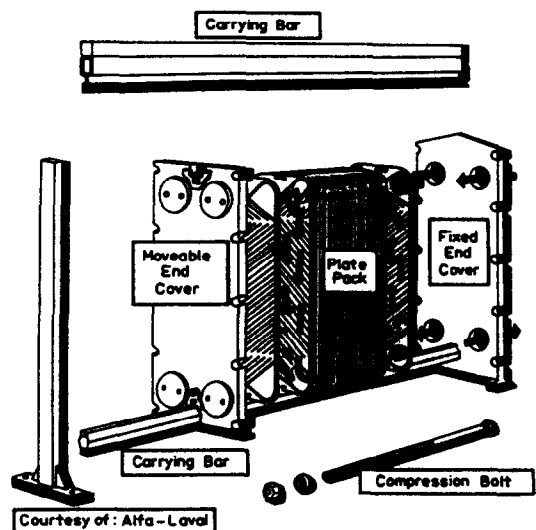


FIG. 1. Plate heat exchanger.

NOMENCLATURE

A	heat transfer area per plate [m ²]	T_i^*	temperature, after transformation, of fluids in channels
A	(8 × 8) tridiagonal matrix with real elements defined in text, equations (2) and (3)	T^*	(8 × 1) column vector of transformed temperatures
C_{pk}	heat capacities of the fluids [J kg ⁻¹ K ⁻¹]	T_0	initial temperature [°C]
$CCTAOUT$	steady state outlet temperature of fluid TA for an equivalent true countercurrent system [°C]	T_0^*	(8 × 1) column vector of transformed initial temperatures
$CCTPOUT$	steady state outlet temperature of fluid TP for an equivalent true countercurrent system [°C]	$TAIN$	inlet temperature of fluid TA [°C]
$ERRA$	percentage error in the outlet temperature of fluid TA	$TAIN1$	inlet temperature of fluid $TA1$ [°C]
$ERRP$	percentage error in the outlet temperature of fluid TP	$TAIN2$	inlet temperature of fluid $TA2$ [°C]
H	eigenvector matrix corresponding to matrix A	$TAOUT$	outlet temperature of fluid TA [°C]
H^{-1}	inverse of the eigenvector matrix	$TAOUT1$	outlet temperature of fluid $TA1$ [°C]
N	number of cells	$TAOUT2$	outlet temperature of fluid $TA2$ [°C]
NC	number of channels in the exchangers	$TPIN$	inlet temperature of fluid TP [°C]
NTU_k	number of transfer units in k th channel, $UA/W_k C_{pk}$	$TPOUT$	outlet temperature of fluid TP [°C]
t	time [s]	U	overall heat transfer coefficient [W m ⁻² K ⁻¹]
Δt	(1/ N)th of the residence time of the fast moving fluid [s]	W	mass flow rate of fluid in channel k [kg s ⁻¹].
T_j	temperature of fluid in channel j of a cell [°C]	Greek symbols	
T	(8 × 1) column vector of temperatures	α_j	$UA/W_k C_{pk} \tau_k$
		τ_k	residence time of fluid in channel k [s].

It can be easily noticed that the problems considered by these authors are steady state performance analyses of plate heat exchangers. Khan *et al.* [7] have performed a frequency response study of a countercurrent plate heat exchanger. They have also mentioned the paucity of studies concerning the dynamics of countercurrent plate heat exchangers. The dynamic analysis of this class of heat exchangers will involve solution of a large system of ordinary differential equations. In some situations this system can have split boundary conditions. Difficulties involved in the numerical solution of problems of this class have been discussed in detail by Lakshmanan and Potter [8]. To overcome these problems, they have developed a new numerical model, namely the 'cinematic' model. This model has been shown to be fast in accurately simulating the dynamic performance of a variety of countercurrent systems in the time domain. It is therefore the objective of this paper to demonstrate the capabilities of the 'cinematic' model in simulating the dynamic behaviour of plate heat exchangers.

APPLICATION OF THE 'CINEMATIC' MODEL

In the following sections of this paper the term plate refers to the partition between the two fluid streams through which heat is transferred from the hot fluid to the cold fluid. Marano and Jechura [6] have considered four types of plate heat exchangers. These are:

- (1) seven plate, two-fluid, looped-flow heat exchanger (configuration A in this work) (Fig. 2);
- (2) five plate, two-fluid, series-flow heat exchanger (Fig. 3);
- (3) five plate, two-fluid, complex-flow heat exchanger (Fig. 4);
- (4) seven plate, three-fluid, looped-flow heat exchanger (configuration B in this work) (Fig. 5).

In the present analysis only two of these four configurations are chosen and they are the seven plate, two-fluid, looped-flow exchanger and the seven plate, three-fluid, looped-flow exchanger. They are chosen to illustrate the application of the 'cinematic' model

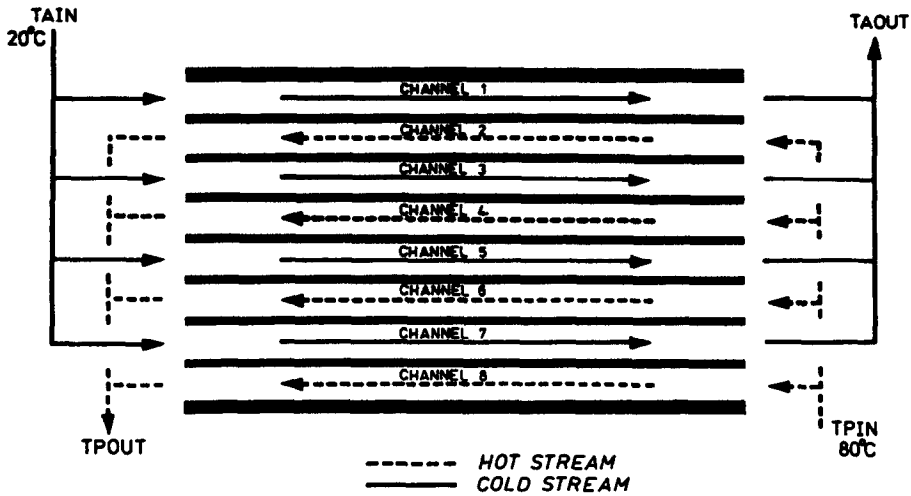


FIG. 2. Seven plate, looped-flow exchanger (configuration A).

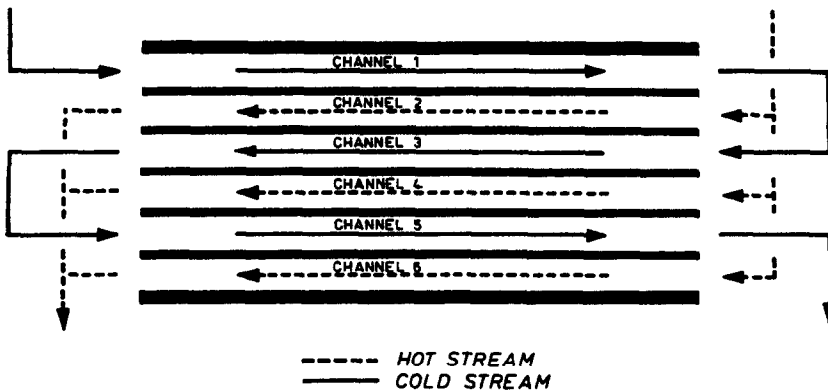


FIG. 3. Five plate, complex-flow exchanger.

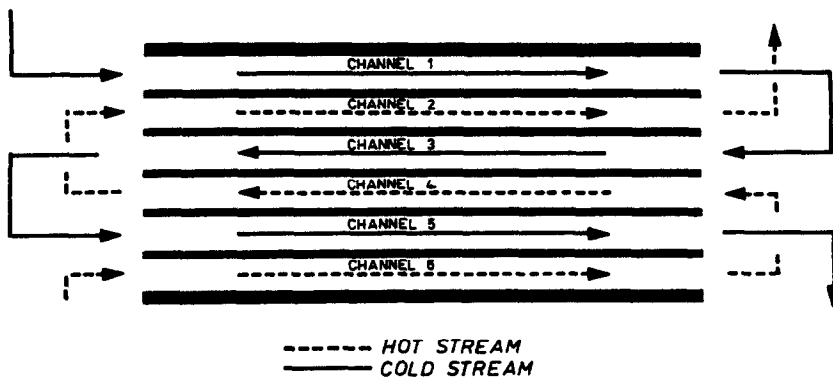


FIG. 4. Five plate, series-flow exchanger.

to problems with different levels of complexity. However, it may be noted that the numerical simulation of different configurations of plate heat exchangers can be performed in an analogous manner.

The following assumptions will be made in applying the 'cinematic' model to plate heat exchangers.

(1) Each fluid is ideally mixed in a direction normal to flow.

(2) Plug flow is assumed due to the very high turbulence in the channels.

(3) Heat conduction in the direction of flow is negligible.

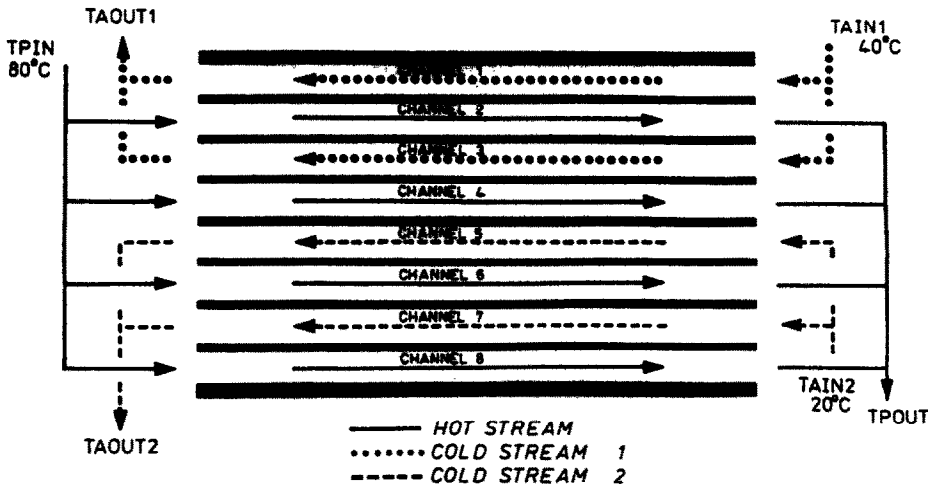


FIG. 5. Seven plate, three-fluid, looped-flow exchanger (configuration B).

(4) Heat is transferred only in the direction normal to the axis of a channel.

(5) An average value is used for the overall heat transfer coefficient.

(6) The effect of temperature on densities and heat capacities of the fluids is neglected.

(7) As the plates are very thin, it is reasonable to assume that they have negligible heat capacitance.

(8) No phase change occurs within the exchanger.

With these assumptions, the equations of the 'cinematic' model will be given next. The flow path between the plates is called a channel. For the seven plate exchanger, there are eight channels. The heat exchanger is divided into N cells. Each cell consists of eight compartments in which fluids flow counter-current to each other. Each fluid is allowed to remain in every cell for $(1/N)$ th of its residence time in a channel. Exchange is allowed in every cell for a differential contact time (Δt) , which is equal to $(1/N)$ th of the residence time of the fast moving stream. After the exchange, the fluid elements are shifted to the next cell in the appropriate direction of flow, based on the ratio of their residence times. Since each compartment is assumed to be well mixed, the exchange process can be written as

$$\frac{dT_1}{dt} = -\alpha_1(T_1 - T_2) \tag{1a}$$

$$\frac{dT_j}{dt} = \alpha_j(T_{j-1} - T_j) - \alpha_j(T_j - T_{j+1}), \quad j = 2-7 \tag{1b}$$

$$\frac{dT_8}{dt} = -\alpha_8(T_7 - T_8). \tag{1c}$$

The coefficients $\alpha_k = UA/W_k C_{pk} \tau_k$ for $k = 1-8$. For the purpose of demonstration of the applicability of the 'cinematic' model, it will be assumed that the heat capacities of the fluid and their mass flow rates in every channel are the same. This will mean that

$\alpha_1 = \alpha_3 = \alpha_5 = \alpha_7$ and $\alpha_2 = \alpha_4 = \alpha_6 = \alpha_8$. Therefore, equations (1a)–(1c) can be written as

$$\frac{dT}{dt} = AT. \tag{2}$$

The elements of the column vector $T(8 \times 1)$ consist of the temperatures of the fluids in eight compartments. The matrix A is an (8×8) real, tridiagonal matrix. The elements of A are given below.

The main diagonal elements are $-\alpha_1, -2\alpha_2, -2\alpha_1, -2\alpha_2, -2\alpha_1, -2\alpha_2, -2\alpha_1$ and $-\alpha_2$. The upper diagonal elements are $\alpha_1, \alpha_2, \alpha_1, \alpha_2, \alpha_1, \alpha_2, \alpha_1$ and α_1 . The lower diagonal elements are $\alpha_2, \alpha_1, \alpha_2, \alpha_1, \alpha_2, \alpha_1$ and α_2 . When $W_1 C_{p1} \tau_1$ is not equal to $W_2 C_{p2} \tau_2$, the temperatures $T_k, k = 1-8$ can be represented by a transformed set of temperatures $T_k^* = \sqrt{(W_1 C_{p1} \tau_1)} T_k$ for $k = 1, 3, 5$ and 7 . For other values of $k, T_k^* = \sqrt{(W_2 C_{p2} \tau_2)} T_k$. With these definitions, the model equations are given by the following set of differential equations:

$$\frac{dT^*}{dt} = AT^*. \tag{3}$$

The matrix A is again an (8×8) real, tridiagonal matrix. Also, it is now a symmetric matrix. This transformation of temperatures is necessary to produce a symmetric tridiagonal matrix A for the case $W_1 C_{p1} \tau_1 \neq W_2 C_{p2} \tau_2$. When $W_1 C_{p1} \tau_1 = W_2 C_{p2} \tau_2$ the matrix A takes the symmetric tridiagonal form even without the transformation. The off-diagonal elements are equal to β where $\beta = \sqrt{(\alpha_1 \alpha_2)}$. The main diagonal elements are $-\alpha_1, -2\alpha_2, -2\alpha_1, -2\alpha_2, -2\alpha_1, -2\alpha_2, -2\alpha_1$ and $-\alpha_2$. Note that $\alpha_1 = NTU_1/\tau_1$ and $\alpha_2 = NTU_2/\tau_2$.

The solution of the system of differential equations (2) is given as $\exp(AT_0)$ and that of equation (3) by $\exp(AT_0^*)$ where T_0 and T_0^* are column vectors containing the initial values of the temperatures of the

two fluids in compartments 1–8 in a general cell. Since it has been shown that the tridiagonal matrix A can be transformed into a symmetric tridiagonal matrix by transforming the temperatures of the fluids, in the following section of this paper only the case of a symmetric tridiagonal system will be considered. To obtain the exponent of a real symmetric tridiagonal matrix, the eigenvalues and the associated eigenvectors of the matrix are to be calculated. Subroutine EIGRF of the IMSL package [9] is used to obtain them. Once the eigenvalues and eigenvectors are made available, the exponent of the matrix is obtained using the equality, $\exp(A\Delta t) = H \exp(\lambda I \Delta t) H^{-1}$ where λI is the tridiagonal matrix of eigenvalues and H the eigenvector. H^{-1} is the inverse of the eigenvector matrix. For a real symmetric tridiagonal matrix, the inverse of the eigenvector matrix is the transpose of the eigenvector matrix. Using this property, $\exp(A\Delta t)$ at $t = \Delta t$ can be calculated and the result of the exchange during this time interval is calculated. The procedures described so far provide the required equations to set up a computer program which will then simulate the dynamic response of a plate heat exchanger.

ALGORITHM

Step 1. Input length of a channel in m, velocity of the fluids, in m s^{-1} , overall heat transfer coefficient in $\text{W m}^{-2} \text{K}^{-1}$, number of channels in the exchanger, heat transfer area per plate in m^2 , heat capacities of the fluids in $\text{J kg}^{-1} \text{K}^{-1}$ and the mass flow rates of the fluids in kg s^{-1} .

Step 2. Input number of cells.

Step 3. Initialize the temperatures of the fluids in all compartments of every cell.

Step 4. Hold each fluid in its compartment for a duration of time Δt , equal to $(1/N)$ th of the residence time of the fast moving stream in a channel.

Step 5. During this period, heat exchange is carried out between the fluids in adjacent compartments. As a result of this exchange the temperatures change and the new values are obtained as described earlier.

Step 6. The fluid elements are moved in the appropriate direction of flow and new quanta are introduced to fill the compartments of cells 1 and N at either end. The temperatures are reinitialized and the calculations repeated from step 5 until steady state is established. The outlet temperatures are also obtained.

Two start-up problems are solved using the 'cinematic' model. The first problem considers a plate exchanger of configuration A and the second one an exchanger of configuration B. The following data is applicable to both systems.

DATA

Length of a channel = 1 m; depth of a channel = 0.005 m; plate width = 0.4 m; velocities of the fluids are equal to 2 m s^{-1} ; overall heat transfer

coefficient = $5000 \text{ W m}^{-2} \text{K}^{-1}$; heat capacities of the fluids are equal to $4187 \text{ J kg}^{-1} \text{K}^{-1}$; heat transfer area per plate = 0.5 m^2 ; mass flow rate of fluids per channel = 4 kg s^{-1} ; densities of fluids are equal to 1000 kg m^{-3} .

PROBLEM 1 (SEE FIG. 2)

Prior to the disturbance the temperatures of the fluids in all compartments are equal to 10°C . At $t = 0$, the inlet temperatures of fluids TA and TP are increased to 20 and 80°C , respectively. The dynamic simulations are performed from this time until the steady state is established.

The averaged outlet temperatures of fluids ($TAOUT$ and $TPOUT$) at steady state are shown as the number of cells are increased in Figs. 6 and 7. It can be seen that only a few cells are required to simulate the dynamics correctly. Additional calculations are performed with the 'cinematic' approach varying the number of channels in the exchanger considered

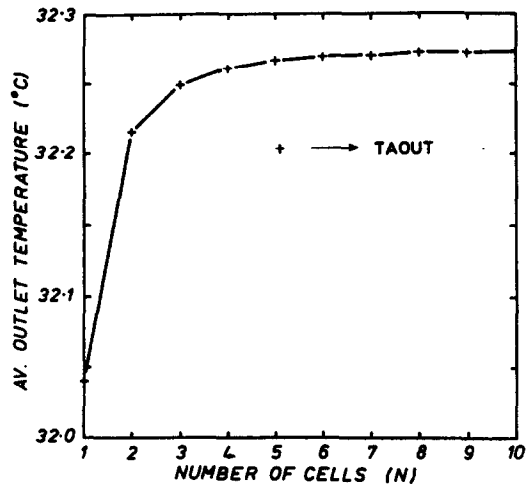


FIG. 6. Effect of number of cells (configuration A).

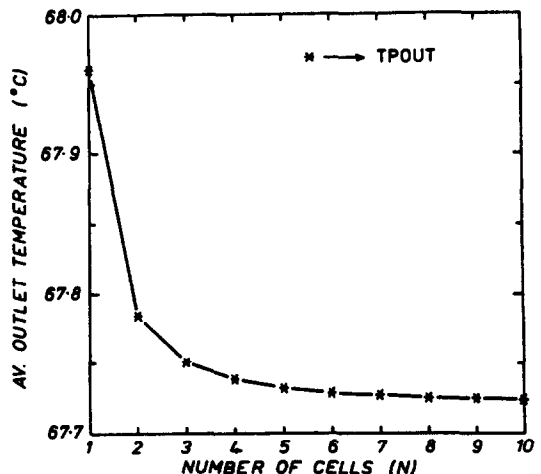


FIG. 7. Effect of number of cells (configuration A).

Table 1. Deviation from the performance of a true countercurrent heat exchanger

NC	NTU	TPOUT	TAOUT	CCTPOUT	CCTAOUT	% ERRP	% ERR
2	0.1493	72.207	27.793	72.2070	27.7930	0.000	0.000
4	0.2239	69.242	30.758	69.0233	30.9767	-0.317	0.706
6	0.2488	68.230	31.770	68.0467	31.9533	-0.269	0.574
8	0.2612	67.724	32.276	67.5728	32.4272	-0.224	0.466
10	0.2687	67.420	32.580	67.2929	32.7071	-0.188	0.389
12	0.2737	67.218	32.782	67.1082	32.8918	-0.164	0.334
14	0.2772	67.073	32.927	66.9771	33.0229	-0.143	0.291
16	0.2799	66.965	33.035	66.8792	33.1208	-0.128	0.259
18	0.2820	66.880	33.120	66.8034	33.1966	-0.115	0.231
20	0.2836	66.813	33.187	66.7430	33.2570	-0.105	0.211

from 2 to 20. The steady state outlet temperatures of the two fluids were compared with the outlet temperatures of an equivalent true countercurrent heat exchanger and the results are shown in Table 1.

For the equivalent true countercurrent system

$$NTU = \frac{UA}{W_1 C_{p1}} \frac{2(NC-1)}{NC}$$

The steady state outlet temperatures are calculated with this value of NTU. The 'per cent' errors are calculated as % deviation of the outlet temperatures of fluids (TAOUT and TPOUT) from the corresponding countercurrent values. From these results it can be seen that a plate heat exchanger with an even number of channels in general does not perform significantly differently from that of an equivalent true countercurrent exchanger. This suggests that quick design calculations can be performed for a plate heat exchanger using the equivalent true countercurrent system as a basis.

PROBLEM 2 (SEE FIG. 5)

Before the starting-up of the heat exchanger, temperatures of the fluids in all channels equal 10°C. At

$t = 0^+$, the inlet temperatures of fluids TAIN1, TAIN2 and TPIN are increased to 40, 20 and 80°C, respectively. The computations are performed from this time until steady state is established. The effect of the number of cells on the averaged outlet temperatures of the fluids is shown in Figs. 8-10. Again, it can be seen

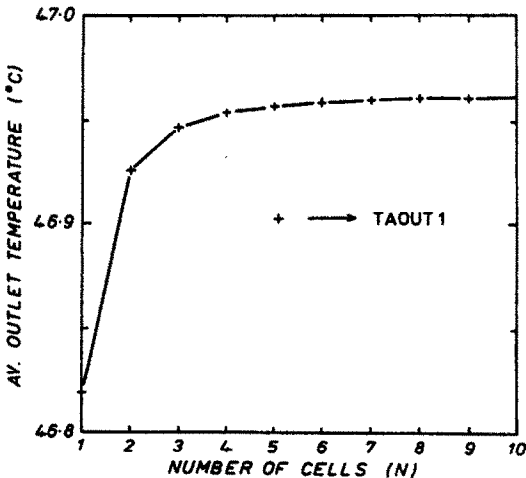


FIG. 8. Effect of number of cells (configuration B).

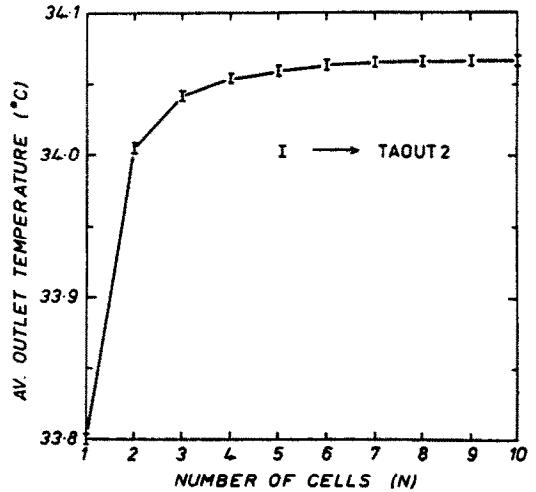


FIG. 9. Effect of number of cells (configuration B).

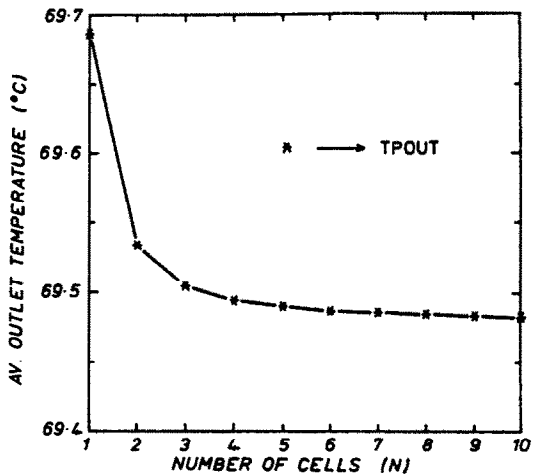


FIG. 10. Effect of number of cells (configuration B).

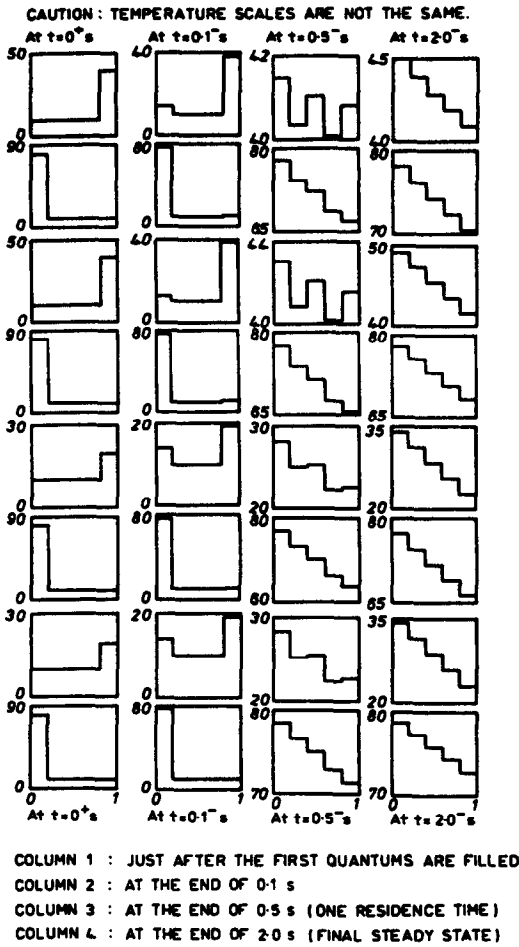


FIG. 11. Dynamic temperature profiles (configuration B).

that a small number of cells is required to simulate the dynamic performance of this type of heat exchanger accurately. Figure 11 shows the dynamic temperature profiles of the fluids from the start-up time. There are

four columns of graphs in this figure. Each column represents the temperature profiles at a particular instant, shown in Fig. 11. Every column contains eight sub-figures representing the eight channels of the exchanger. It must be noted that the temperature scales are not the same in Fig. 11. The averaged values of the outlet temperatures of fluids (*TPOUT*, *TAOUT1* and *TAOUT2*) are plotted against dimensionless time (time/residence time in a channel) in Fig. 12. This indicates that the exchanger is close to steady state operating conditions at a dimensionless time equal to 2.

DISCUSSION AND CONCLUSION

The 'cinematic' model has been applied to evaluate the dynamic performance of two types of plate heat exchangers. The procedures indicated here can be applied to other types of plate heat exchangers. Integral ratios of residence times are chosen for easier demonstration of the ability of the model. However, for other values procedures similar to the one indicated can be adopted. Also, varying heat transfer coefficients and any other non-linearity can be adopted easily while similar attempts using methods such as Laplace transforms may prove to be very difficult computational problems. It has also been shown that the 'cinematic' model solves the design problem which requires just the steady state temperature profiles and the performance problem which requires the dynamic behaviour of the exchanger. If the computational efforts involved in implementing this technique on a digital computer are compared to the previously reported techniques, two important points can be summarized. The first one is that the 'cinematic' model requires the least amount of calculations to provide accurate and fast results due to the simple algorithm. Secondly, the 'cinematic' model provides not only the steady state temperature profiles

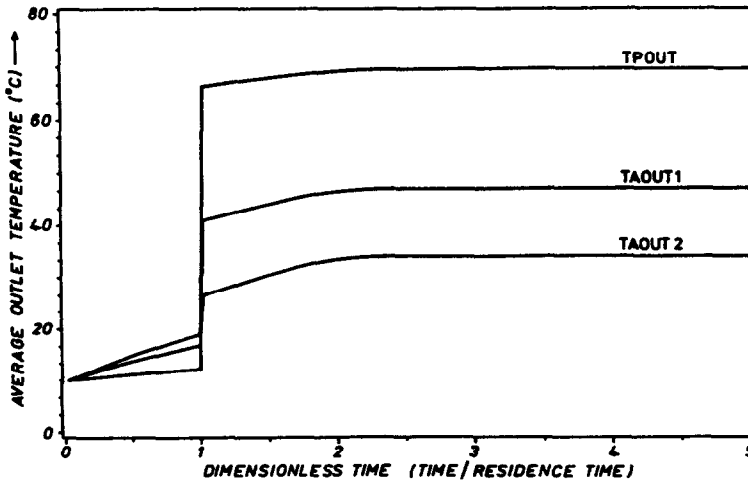


FIG. 12. Approach to steady state (configuration B).

but also the dynamic temperature profiles, thus solving the design and the performance problems in one sweep. Further applications of this model are in progress.

REFERENCES

1. E. L. Watson, A. A. McKillop, W. L. Dunkley and R. L. Perry, Plate heat exchanger-flow characteristics, *Ind. Engng Chem.* **52**(9), 733-744 (1960).
2. B. W. Jackson and R. A. Troupe, Plate heat exchanger design by ϵ -NTU method, *Chem. Engng Prog. Symp. Ser.* **62**(64), 185-190 (1966).
3. R. A. Buonopane, R. A. Troupe and J. C. Morgan, Heat transfer design method for plate heat exchangers, *Chem. Engng Prog.* **59**(7), 57-61 (1963).
4. J. Wolf, General solution of the equations of parallel flow multichannel heat exchangers, *Int. J. Heat Mass Transfer* **7**, 901 (1964).
5. T. Zaleski, A general mathematical model of parallel-flow, multichannel heat exchangers and analysis of its properties, *Chem. Engng Sci.* **39**, 1251-1260 (1984).
6. J. J. Marano and J. L. Jechura, Analysis of heat transfer in plate heat exchangers, *A.I.Ch.E. Symp. Ser.* **81**(245), 116-121 (1985).
7. A. R. Khan, N. S. Baker and A. P. Wardle, The dynamic characteristics of a countercurrent plate heat exchanger, *Int. J. Heat Mass Transfer* **31**, 1269-1278 (1988).
8. C. Lakshmanan and O. E. Potter, Dynamic simulation of heat exchangers and fluidised beds, *Proc. 12th Aust. Chem. Engng Conf. CHEMECA 84*, Vol. 2, pp. 871-878 (1984).
9. *IMSL International Mathematical and Statistical Library*. IMSL Inc., Houston, Texas (1982).

SIMULATION DYNAMIQUE DES ECHANGEURS THERMIQUES A PLAQUES

Résumé—Des modèles précédemment publiés concernent la performance de régime permanent des échangeurs thermiques à plaques en utilisant la solution numérique des équations différentielles qui décrivent le transfert de chaleur. Le nouveau modèle numérique, dit modèle "cinématique", développé par les auteurs a été utilisé pour simuler le comportement variable des systèmes à contre-courant tels que les lits fluidisés (C. Lakshmanan et O. E. Potter, *Ind. Engng Chem. Res.* **26**, 292-296 (1987)). On traite ici de l'application de ce modèle à la simulation de la performance dynamique des échangeurs thermiques à plaques. On montre que le modèle "cinématique" nécessite un très faible temps de calcul en donnant une simulation précise. Il calcule aussi les profils variables et stationnaires, offrant ainsi une approche aisée et précise pour les problèmes de conception considérés par d'autres modèles. Il montre que représenter un échangeur à plaques avec un certain nombre de canaux par un système à contre-courant équivalent, ne conduit pas à des erreurs significatives quant on considère les températures de sortie en régime permanent.

DYNAMISCHE SIMULATION VON PLATTENWÄRMETAUSCHERN

Zusammenfassung—Modelle, die bisher in der Literatur beschrieben wurden, sind in der Lage, das stationäre Verhalten von Plattenwärmetauschern zu berechnen. Dabei werden die Differentialgleichungen, welche die Wärmeübertragung beschreiben, numerisch gelöst. Das neue, sogenannte "kinematische" Modell, das von den Autoren entwickelt wurde, ist in der Lage, das dynamische Verhalten von Gegenstromsystemen, wie z. B. Wirbelbetten, zu simulieren (C. Lakshmanan and O. E. Potter, *Ind. Engng Chem. Res.* **26**, 292-296 (1987)). In der vorliegenden Arbeit wird die Anwendung des kinematischen Modells bei der Simulation des dynamischen Verhaltens von Plattenwärmetauschern gezeigt. Das kinematische Modell benötigt außerordentlich wenig Berechnungen, um die Dynamik von Plattenwärmetauschern exakt zu beschreiben. Die dynamischen und stationären Profile werden in einem Durchgang berechnet; dadurch steht für Auslegungsprobleme ein einfaches und genaues Verfahren zur Verfügung. Es zeigt sich auch, daß bei einem Plattenwärmeübertrager mit einer geraden Zahl von Kanälen keine spürbaren Fehler bei der Berechnung der stationären Austrittstemperaturen entstehen.

ДИНАМИЧЕСКОЕ МОДЕЛИРОВАНИЕ ПЛАСТИНЧАТЫХ ТЕПЛОБМЕННИКОВ

Аннотация—Ранее предложенные в литературе модели предназначены для определения стационарного режима работы пластинчатых теплообменников на основе численного решения дифференциальных уравнений, описывающих процесс теплопереноса. Новая численная модель, а именно, разработанная авторами данной работы модель "Синематик", используется для моделирования динамического поведения таких противоточных систем, как псевдоожиженные слои. Рассматривается применение модели "Синематик" для моделирования динамического режима пластинчатых теплообменников. Показано, что для точного моделирования динамики пластинчатых теплообменников с использованием данной модели требуется минимальное количество расчетов. Модель "Синематик" позволяет также одновременно рассчитывать динамические и стационарные профили. Этим методом можно более просто и точно решить задачи, исследованные другими авторами. Показано, что представление пластинчатого теплообменника с четным количеством каналов эквивалентной противоточной системой не приводит к значительным погрешностям определения стационарных температур на выходе.